(6 pages)

Reg. No. : ....

Code No. : 30342 E **Sub. Code : JMMA 63/ JMMC 63**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2020.

Sixth Semester

Mathematics/Mathematics with CA – Main

NUMBER THEORY

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Every non empty set S of nonnegative integers contains a ———— element.

   (a) least (b) greatest

   (c) zero (d) infinity

2. $\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\dots=$

(a) 2^n (b) 2^{n-1}

(c) 2^{n+1} (d) 0

3. If t_n is the nth triangular number, then $t_n =$

(a) $\binom{n}{2}$ (b) $\binom{n-1}{2}$

(c) $\frac{n+1}{2}$ (d) $\binom{n+1}{2}$

4. $\text{lcm}\,(a,b)=ab$ if and only if ________.

(a) $\gcd(a,b)=1$ (b) $\gcd(a,b)=a$

(c) $\gcd(a,b)=b$ (d) $\gcd(a,b)=ab$

5. $5^{\#}+1=$ ________.

(a) 5 (b) 6

(c) 11 (d) 31

6. The canonical form of 360 is ________.

(a) 300 + 60 + 0 (b) 3 + 6 + 0

(c) $5\times 8\times 9$ (d) $2^3.3^2.5$

7. The remainder when we divide $1!+2!+...+100!$ by 12 is ———.

(a) 0 (b) 9

(c) 11 (d) 1

8. $-15 \equiv$ ——— $(\bmod 7)$.

(a) 64 (b) –20

(c) –64 (d) 0

9. If p and q are distinct primes with $a^p \equiv a (\bmod q)$ and $a^q \equiv a (\bmod p)$, then $a^{pq} \equiv$ ——— $(\bmod pq)$.

(a) a^2 (b) 1

(c) a (d) 0

10. The least pseudoprime is ———.

(a) 2 (b) 101

(c) 341 (d) 1001

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the first principle of finite induction.

Or

(b) State and prove Pascal's rule.

12. (a) Prove that $a(a^2+2)/3$ is an integer for all $a \geq 1$.

Or

(b) Find g.c.d. (12378, 3054) using Euclidean algorithm.

13. (a) Show that the number of primes is infinite.

Or

(b) If p_n is the nth prime number, prove that $p_n \leq 2^{2^{n-1}}$.

14. (a) If $ca \equiv cb(\bmod n)$, then prove that $a \equiv b\left(\bmod \frac{n}{d}\right)$, where $d = \gcd(c,n)$.

Or

(b) Solve the linear congruence $9x \equiv 21(\bmod 30)$.

15. (a) State and prove Fermat's theorem.

Or

(b) Factorize the number 12499.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) State and prove the Archimedean Property.

(ii) Derive Newton's identity.

Or

(b) State and prove the binomial theorem.

17. (a) Given integers a and b, not both of which are zero, show that there exist integers x and y such that $\gcd(a,b) = ax + by$.

Or

(b) A customer bought a dozen pieces of fruit, 12 apples and oranges for Rs. 132. If an apple costs Rs. 3 more than an orange and more apples than oranges were purchased, how many pieces of each kind were bought?

18. (a) State and prove the fundamental theorem of arithmetic.

Or

(b) (i) Prove that $\sqrt{2}$ is irrational.

(ii) Show that there are an infinite number of primes of the form $4n+3$.

19. (a) Prove that the linear congruence $ax \equiv b \pmod{n}$ has a solution if and only if $d | b$ where $d = \gcd(a, n)$. If $d | b$, then it has d mutually incongruent solutions modulo n.

Or

(b) State and prove Chinese reminder theorem.

20. (a) Let p be an odd prime. Prove that the quadratic congruence $x^2 + 1 = 0 \pmod{p}$ has a solution if and only if $p \equiv 1 \pmod{4}$.

Or

(b) State and prove Wilson's theorem.